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Mathematical Model of ^{137}Cs Vertical Migration in a Forest Soil

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ABSTRACT

A detailed simulation model for prediction of the magnitude and direction of radionuclide migration in forest soils is described. The mathematical model has been constructed to simulate ^{137}Cs dynamics in the soddy podzolic soils of a mixed forest system (predominantly oak with some birch and pine) in the 30-km zone around the Chernobyl Nuclear Power Plant (ChNPP). Field data gathered by the Radioecology Laboratory of Moscow State University were used both for parameterization of the model and also for validation. Some problems encountered in the mathematical modeling of ^{137}Cs migration in these ecosystems are discussed.

INTRODUCTION

There are a number of mathematical models describing radionuclide migration in both idealized, hypothetical 'soils' and in soils of agricultural ecosystems (Prokhorov, 1970, 1981; Pegoyev, 1979; Billyi *et al.*, 1992; Loshilov *et al.*, 1992; Crout Neil *et al.*, 1990). However, the extreme heterogeneity of forest ecosystems causes serious problems in mathematical modeling and there are rather few models of such ecosystems.

Several current models of radionuclide migration in forest ecosystems pay little attention to vertical migration in the soil, for example, the model describing ^{90}Sr behaviour, developed by Alexakhin *et al.* (1976) or Radforet (Van Voris *et al.*, 1990). The model RABES (Velasco *et al.*, 1993) was developed to study migration of radionuclides in soils of natural grasslands and beech woods, and is rather more realistic, but the time step used in the model is too large for application to heterogeneous forest ecosystems. Moreover, it is not possible to take into account seasonal variations, or any second contamination event. The model described here is intended to remedy these weaknesses.

DESCRIPTION OF THE MODEL

The flow diagram presented in Fig. 1 shows the topological structure of the model. Blocks represent the ^{137}Cs content of the forest litter and of the underlying, so-called 'mineral' horizons. Solid lines are migration pathways, dotted lines are the boundaries of the soil profile, and valves represent transfer functions.

The model uses the following variables to describe the radionuclide content of the soil: *Sao*—global radionuclide content in the forest litter $(A_0L + A_0F + A_0H)$; $h(i)$—mobile component of radionuclides in the lower layers of the soil; and $Y(i)$—immovable component, where $i = 1, \ldots, n$ is the number of soil layers.

The distributive pool (R) is used only to describe radionuclide redistribution in the soil by roots and fungal hyphae. The model comprises the following differential equations:

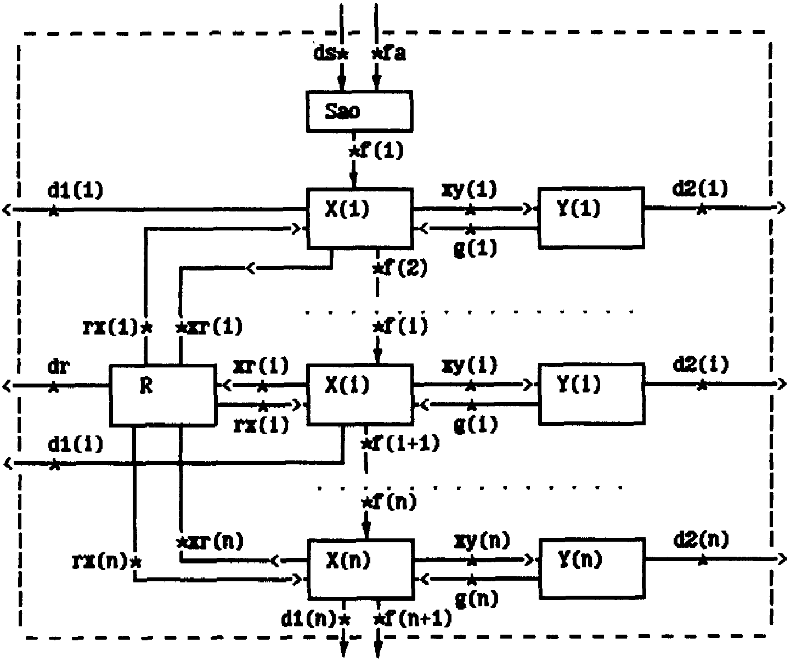


Fig. 1. Flow diagram of the vertical radionuclide migration in the soil.

$$\mathrm{d}Sao/\mathrm{d}t = fa - f(1) - ds \tag{1}$$

$$\mathrm{d}X(i)/\mathrm{d}t = f(i) - f(i+1) - d1(i) - xy(i) + g(i) + rx(i) - xr(i) \tag{2}$$

$$\mathrm{d}Y(i)/\mathrm{d}t = xy(i) - d2(i) - g(i) + df(i) - df(i+1) \tag{3}$$

$$\mathrm{d}R/\mathrm{d}t = xr(i) - rx(i) - dr \tag{4}$$

The time step used is 1 day and the thickness of the individual soil layers under forest litter is 1 cm.

A computer program to perform the model calculations has been written in QuickBASIC, with the results being output in graphic form (see Fig. 2 for an example).

TRANSFER FUNCTIONS

The entrance of radionuclides into the soil both directly from fallout and indirectly from litter fall is described by fa. As the model is presently constructed, the Chernobyl fallout of spring 1986 is simulated.

$$fa = a_1 \text{ on the day of the accident, and } fa = 0, \text{ thereafter} \tag{5}$$

$f(1)$ is the outflow of radionuclides from litter due to litter mineralization and leaching from radioactive particles and is given as

$$f(1) = a_2 \times \mathrm{Sao} \tag{6}$$

$f(i)$ is the downward migration of radionuclides in the soil (convection), given by

$$f(i) = a_3 X(i) \tag{7}$$

Diffusion, $\mathrm{d}f(i)$ is described by

$$df(i) = d(y(i) - y(i+1))/2 \tag{8}$$

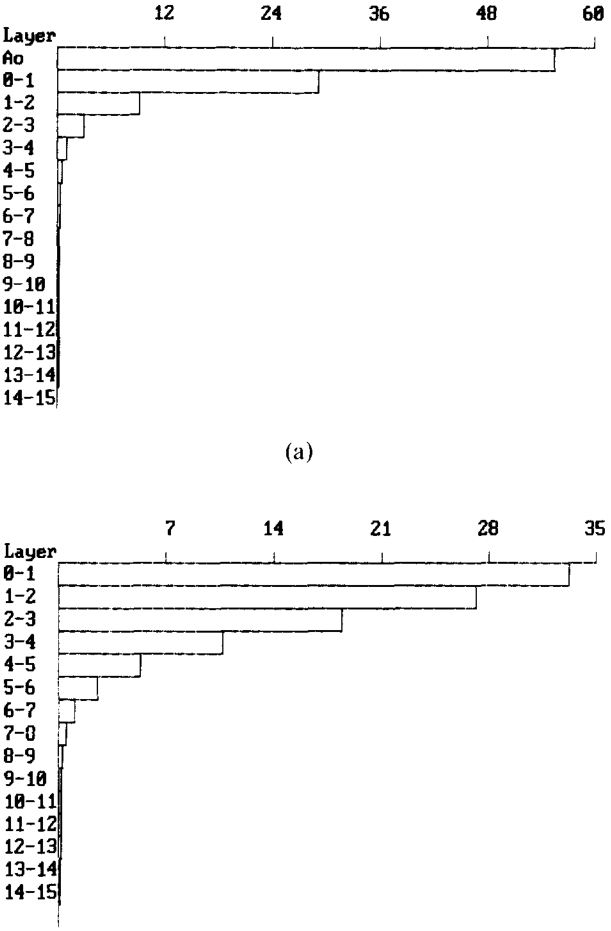
and radioactive decay in the corresponding blocks, ds, $d1(i)$, $d2(i)$, dr is given by

$$ds = a_4\mathrm{Sao},\ \ d1(i) = a_4X(i),\ \ d2(i) = a_4Y(i),\ \ dr = a_4R \tag{9}$$

The fixation of the mobile component in layer i due to adsorption, ion exchange and precipitation in organic and organo-mineral complexes, $xy(i)$, is given by

$$xy(i) = a_5X(i) \tag{10}$$

and the transition of the immovable component to the mobile state as a result of desorption, ion exchange and/or redissolution, $g(i)$ is



(a)

(b)

Fig. 2. (a) Forest litter (A_0) have variable thickness, the thickness of the lower layers is equal to 1 cm. (b) The situation of the forest litter absence is imitated, the thickness of the layers is equal to 1 cm.

$$g(i) = a_6 si(i) Y(i) \tag{11}$$

where *si* is an auxiliary variable reflecting the sigmoid variation in soil exchange capacity with depth (i) and the thickness of the humus horizon (hc)

$$si(i) = (i-1)^2/((((hc-1)0{\cdot}5-1)^2-1)+(i-1)^2) \tag{12}$$

Radionuclide uptake by root systems is often not taken into account when modeling migration in soil, since the vegetation generally accounts for only a small fraction of the radionuclide inventory. However, chemical elements present in ionic form are relatively readily taken up and translocated by plant roots, so that transport processes within plant tissues may result in downward transport of small quantities of radionuclides, comparable to those in the plant cover.

The absorption of radionuclides by roots and fungus filaments, $xr(i)$, is given in eqn (13). It is assumed that only the previously defined mobile component is biologically available.

$$xr(i) = a_7 X(i) \tag{13}$$

The release of radionuclides due to turnover and biological activity or root and fungal tissues, $rx(i)$, is

$$rx(i) = a_8 Rrd(i) \tag{14}$$

where rd is an auxiliary variable describing distribution of roots and fungal hyphae in the soil profile. In the current version of the model, the distribution is constant in time, but decreases from 40% in the uppermost layer to 0·5% in the deepest.

PARAMETERS

In Table 1, the parameters used in the model are summarized. Unfortunately, in many cases, there are either no values of the required coefficients in the literature, or else the published data cover an unacceptably

TABLE 1
Parameters of the Model

Value	*Units*	*Definition*
a_1 User determined	kBq/m^2	Quantity of radionuclide fallout on this territory in 1986
a_2 0·000095	per day	Coefficient of the radionuclide entrance from the litter
a_3 0·02	per day	Coefficient of the radionuclide migration from upper to lower layer of the soil
a_4 0·0000633	per day	Constant of the radioactive decay
a_5 0·01	per day	Coefficient of the mobile component fixation
a_6 0·00005	per day	Coefficient of the immovable component transition into the mobile state
a_7 0·035	per day	Part of the radionuclides absorbed by roots and fungus filaments from present soil layer
a_8 0·2	per day	Coefficient of the radionuclide release from roots and mycelium
d 0·000864	per day	Diffusion coefficient

wide range and can be used only to provide approximate values. Where these problems were encountered, the best value of the required parameter was obtained by iterative fitting of field and literature data. In general the field data used were obtained from studies carried out by the Radioecology Laboratory (Soil Science Faculty, Moscow State University). These investigations were conducted within the ChNPP 30-km zone between 1986 and 1992 and results have already been published (Tikhomirov *et al.*, 1990; ECP-5 Report, 1992; Shcheglov *et al.*, 1992). These studies provided information about the relative radionuclide distributions in the soil profile and also about the cycling of the radionuclides in the soil. Some of the model parameters are derived from lysimeter studies in soils of the zone (Tikhomirov *et al.*, 1992).

VALIDATION OF THE MODEL

Model validation was carried out using field data for the ^{137}Cs distribution in the soil profile. Figure 3 illustrates the recycling of ^{137}Cs in the litter. It should be noted that data taken in the last three years (1990–1992) are believed to be better since a more reliable method of soil sampling was used in this period. This said, agreement between field data and model results is generally good. For example, in Table 2, field observations and model calculations of the ^{137}Cs profile distributions in 1990–1992 are presented and it is clear that agreement between field data and computed data is satisfactory.

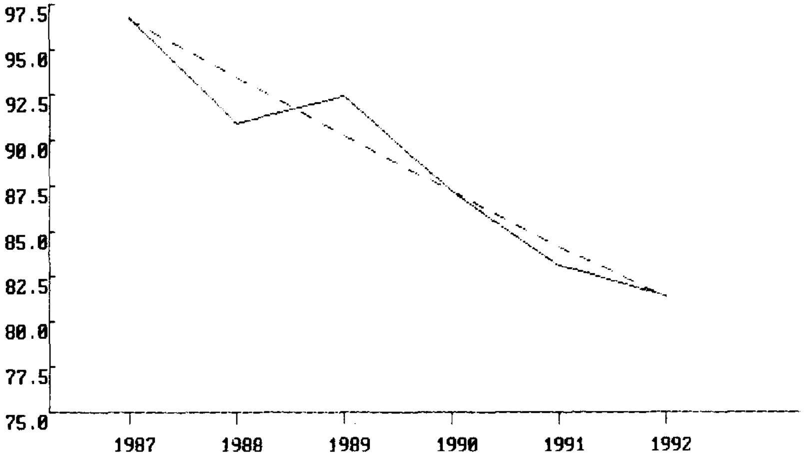


Fig. 3. Dynamics of ^{137}Cs content in the forest litter (A_0) percentage of total amount of ^{137}Cs in the soil. —— Field data; – – – computed results.

TABLE 2
Dynamics of ^{137}Cs Distribution in the Soddy Podzolic Sandy Soil Profile Percentage of Total Content

Layer (cm)	*Field data*			*Computing results*		
	1990	*1991*	*1992*	*1990*	*1991*	*1992*
A_0	87·26	83·1	81·4	87·19	84·22	81·35
0–1	5·0	8·5	12·04	8·44	10·39	12·28
1–2	2·08	3·5	4·09	2·62	3·23	3·82
2–3	0·84	1·8	1·35	0·84	1·04	1·23
3–4	0·57	1·3	0·52	0·30	0·38	0·45
4–5	0·47	0·8	0·26	0·14	0·17	0·20
5–6	0·4	—	0·08	0·09	0·11	0·13
6–7	0·41	0·4		0·07	0·08	0·10
7–8	0·3	—	0·09	0·06	0·08	0·09
8–9	0·23	0·3		0·05	0·06	0·08
9–10	0·15	—	0·14	0·05	0·06	0·07
10–11	0·15	0·3		0·04	0·05	0·06
11–12	0·07	—	—	0·04	0·05	0·06
12–13	—	—	—	0·03	0·03	0·04
13–14	—	—	—	0·02	0·03	0·03
14–15	—	—	—	0·01	0·02	0·02

RESULTS AND DISCUSSION

The model has been used to make predictions of the ^{137}Cs profile distributions. Data concerning radionuclide dynamics have also been calculated and are presented in Table 3. These results support the suggestion that forest litter functions as an effective biogeochemical barrier to the downward migration of radionuclides. Figure 2(b) illustrates the increase in downward migration rate which is predicted to occur in the absence of a litter layer.

Sensitivity studies have shown that the model output is particularly affected by the following parameters: the release of the radionuclide from the litter layer (a_2); radionuclide migration from upper to lower layer of the soil (a_3); the thickness of the humic horizon (hc). As a result, the model is only of limited use in making more general predictions and its use should be confined to the description of soil migration processes in ecosystems similar to those for which the model was developed.

The reliability of model predictions is dependent on the limitations of the individual model. The particular merits of this model are its simplicity and the possibility of predicting radionuclide depth distributions, starting only with information on the average density of radioactive contamina-

TABLE 3

Prognosis of ^{137}Cs Distribution Dynamics in the Profile of the Soddy Podzolic Sandy Soil, Percentage of Total Content in the Soil

Year:	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*
A_0	78·58	75·90	73·32	70·82	68·41	66·08	63·82	61·65	59·55	57·52	55·56
0–1	14·07	15·82	17·51	19·15	20·73	22·25	23·73	25·15	26·52	27·85	29·13
1–2	4·39	4·94	5·47	5·98	6·48	6·96	7·42	7·86	8·30	8·71	9·12
2–3	1·42	1·60	1·78	1·94	2·10	2·26	2·41	2·56	2·70	2·84	2·97
3–4	0·52	0·58	0·64	0·71	0·76	0·82	0·88	0·93	0·98	1·03	1·08
4–5	0·24	0·27	0·30	0·33	0·35	0·38	0·40	0·43	0·45	0·48	0·50
5–6	0·15	0·17	0·19	0·21	0·23	0·24	0·26	0·28	0·29	0·31	0·32
6–7	0·12	0·13	0·14	0·16	0·17	0·18	0·20	0·21	0·22	0·23	0·24
7–8	0·10	0·12	0·13	0·14	0·15	0·17	0·18	0·19	0·20	0·21	0·22
8–9	0·09	0·10	0·11	0·12	0·13	0·14	0·15	0·16	0·17	0·18	0·19
9–10	0·08	0·10	0·11	0·12	0·13	0·13	0·14	0·15	0·16	0·17	0·18
10–11	0·07	0·08	0·09	0·10	0·11	0·11	0·12	0·13	0·14	0·14	0·15
11–12	0·07	0·08	0·08	0·09	0·10	0·11	0·11	0·12	0·13	0·14	0·14
12–13	0·04	0·05	0·05	0·06	0·06	0·07	0·07	0·08	0·08	0·09	0·09
13–14	0·04	0·04	0·05	0·05	0·05	0·06	0·06	0·07	0·07	0·07	0·08
14–15	0·02	0·03	0·03	0·03	0·03	0·04	0·04	0·04	0·04	0·05	0·05

tion. It is, however, important that the collection of soil cores for detailed analysis required for this model is very difficult. The accuracy and skill of the investigator are important, but also field data may vary because of soil cover heterogeneity (microrelief, species composition, productivity of plant cover, thickness of litter layer, soil turnover by burrowing animals etc.) and uneven, 'mosaic' fallout. To obtain a realistic picture, it is therefore necessary to analyse a large number of individual samples. The mathematical model described here, however, permits simulation of the soil processes with only a minimum of information.

Shortcomings in the model include the absence of any effects of soil temperature and hydrology, or of seasonal vegetation dynamics. The model will be most reliable for making predictions in the first few years after an accident, but can also be used to make a more approximate estimate of future developments. In the short-term, it is proposed to address some of the shortcomings identified in the current model by linking it with a model of seasonal element dynamics in a similar system, which already exists (Mamikhin, 1987, 1990). This approach would significantly increase the degree of realism in the radionuclide model. It would also be useful to include more detailed descriptions of the sorption of radionuclides by soil organic matter, soil minerals and microfauna; all factors which recent field studies (Agapkina *et al.*, 1991; Thiry *et al.*, 1993) have identified as being important.

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